Edition

2012 edition. This has been updated to the 10^{th} edition of the **ARRL Extra Class License Manual**.

Approach

This presentation covers the material at three levels:

1. just enough to be able to identify and answer the questions found in the Extra class examination,
2. enough to work most problems encountered while building and operating a station and
3. enough to understand what the mathematics is doing. Calculus, matrices and other advanced mathematics are used at this level.
   No advanced mathematics is needed for the Extra Class test!

Please, select those levels of interest to you.
The Extra class test is limited by what is practical to compute in a quarter hour using a basic calculator. Only at the end of these notes is there material beyond what strictly is needed for the Extra class test.

Introduction

When working with alternating current (AC) and reactive components, it is necessary to work with complex arithmetic. Complex values may be presented in either rectangular or polar coordinates; i.e., so much left or right and so much up or down or that direction a certain distance. In most of the formulae below, the formulae use complex arithmetic for AC and reactive components and real (ordinary) arithmetic for direct current (DC) or only resistive components.
To work with angles requires some knowlege of trigonometry. If complex arithmetic is available, that is an advantage; but with rectangular to polar and polar to rectangular coordinate conversion, complex multiplication and division are not difficult (see the last item in Shortcuts).

Desirable equipment

For the first level, a four-function calculator: add, subtract, multiply and divide, possibly with percent (%) or square root ($\sqrt{x}$). Such calculators easily can be found at dollar stores.
For the other levels, a low-end scientific calculator:

1. add, subtract, multiply and divide,
2. the trigonometric functions and the inverse trigonometric functions (.i.e., look up the angle for the function value);
3. raising to a power (x^y), logarithms base e and 10 and raising e and 10 to a power;
4. rectangular to polar and vis-a-vis conversions (this can be done with the trigonometric functions, it is much easier with these built-in) and
5. complex arithmetic is desirable but not necessary,

Calculators with memory or formulae storage must be cleared and checked to be cleared by the Volunteer Examiners. Calculators with built-in formulae or unit conversions or are programmable generally are prohibited. This includes most HP calculators and the high end Canon, Casio, Sharp and TI calculators. The only HP models that are not programmable as of Nov, 2011, are the single-digit models. The Canon F-710 has the resonant circuit frequency formula built in.

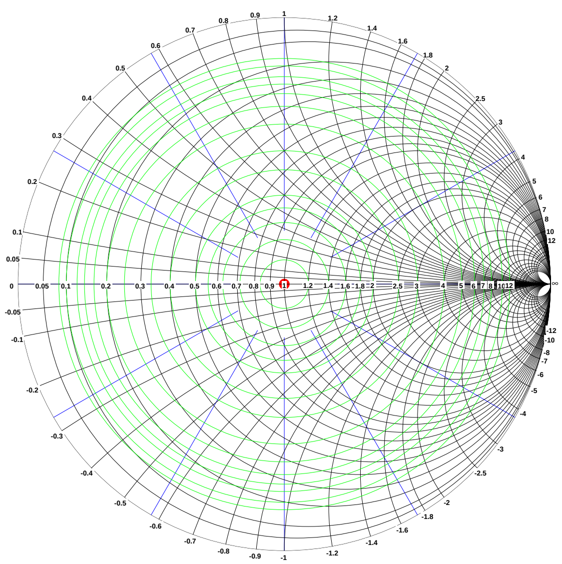
Shortcuts to the correct answers

1. A simple Smith chart comes in the graphics section of the Extra test. As presented there,
   (a) 0Ω or *shorted*, which can recognized by the wide spacing of the resistance circles, is at the top (12 o'clock),
   (b) $1\jmath\Omega$ or *inductive*, is at the right (3 o'clock),
   (c) $\infty\Omega$ or *open*, which can recognized by the narrow spacing of the resistance circles, is at the bottom (6 o'clock) and
   (d) $1\jmath\Omega$ or *capacitive*, is at the left (9 o'clock).
2. This orientation is rotated $90°$ clockwise from the usual orientation of the nomograph, which has the resistance axis horizonal.
3. The standard resistance, e.g., 50Ω is placed at the center of the chart. The complete circles tangent to the $\infty\Omega$ end of the resistance axis are resistance circles. The partial arcs with one end at the $\infty\Omega$ end of the resistance axis are reactance (inductive and capacitive) arcs. Circles around the center of the chart are SWR circles. The SWR ratio is the value on the resistance axis where the SWR circle crosses the resistance axis between the center and infinite resistance. The units of impedance are normalized to "standard" value at the center of the chart. The "standard" value is usually the impedance of the feedline. In the of a 50Ω cable, that 50Ω multiplies all of the impedance values on the Smith chart.
   Learn your way around the Smith chart. Reread and understand the material on the Smith chart above. This will answer the E9G questions, including the question on SWR circles. Remember that distance towards the generator (transmitter) is clockwise on the outside edge of the chart.
   The outer edge is measured in transmission line wavelength units of length. It is a half-wave length around the entire outside. That length is measured one-way; even though the signal goes round-trip. For example, any signal reflected by the far end back to the generator (transmitter) is delayed by

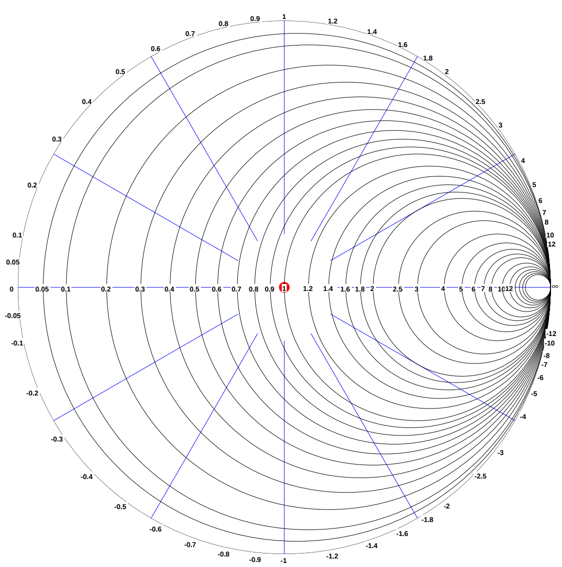
twice the transmission line wavelength length: $\frac{3}{8}\lambda$ delays a signal by $\frac{3}{4}\lambda$ by the time of its return to the generator (transmitter). The generator sees the sum of its output and the returned signal.
That will answer all of the section E9F questions about open and shorted (at the far end) transmission lines of $\frac{1}{8}\lambda$, $\frac{1}{4}\lambda$, $\frac{3}{8}\lambda$ and $\frac{1}{2}\lambda$.

far end is:	$\frac{1}{8}\lambda$	$\frac{1}{4}\lambda$	$\frac{3}{8}\lambda$	$\frac{1}{2}\lambda$
shorted (S)	L	O	C	S
inductive (L)	O	C	S	L
open (O)	C	S	L	O
capacitive (C)	S	L	O	C

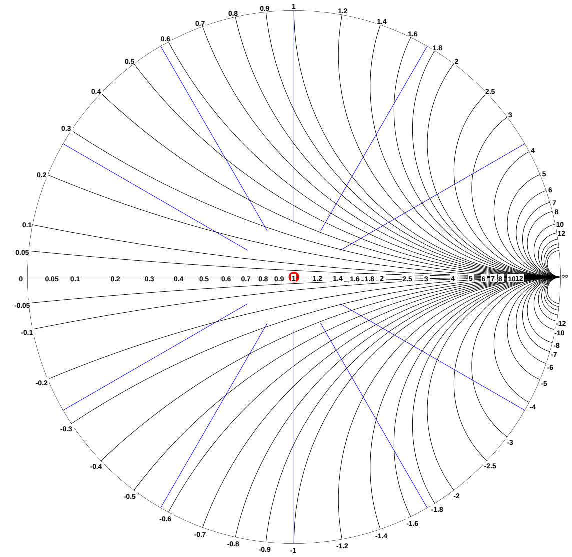
Remember: double the λ distance to get the distance around the full circle.



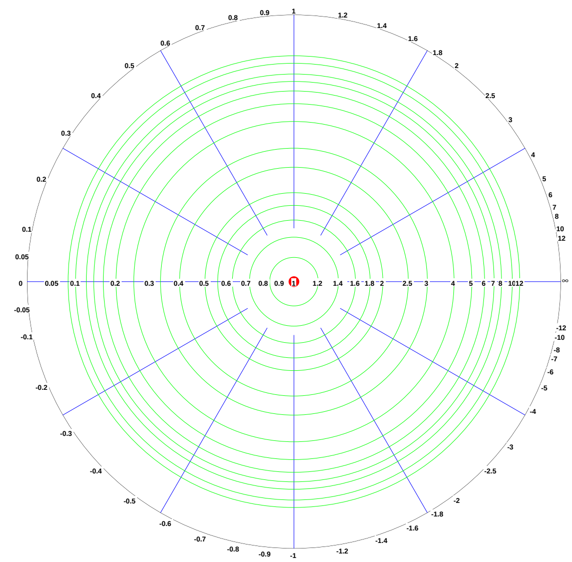
Here is the complete Smith chart. Below, it is broken down into its major parts with the labeling and phase angle bars to provide a reference frame.



Above is the Smith chart with its framing and resistive circles.



Above is the Smith chart with its framing and reactance arcs.



Above is the Smith chart with its framing and standing wave ratio circles.

4. For the questions on two adjacent $\frac{1}{2}\lambda$ vertical dipole antennae fed with a specified spacing and phasing:

	0°	90°	180°
$\frac{1}{4}$		cardiod	
$\frac{1}{2}$	broadside		end fire

In the 2008 Extra test, there are only three questions on this topic. The phasing is enough to tell them apart.

5. Definitions of AC voltages:

(a) Root-mean-square (RMS):

$$\sqrt{\frac{\int_0^{2\pi} v(t)^2 dt}{2\pi}} = \sqrt{\int_0^1 v(t)^2 dt}$$

This is the square root of the arithmetic average, over a complete cycle of the squares of the values. This is the normal way of measuring AC voltage. For $v(t) = \sin(t)$, this is $\sqrt{\frac{1}{2}}$ times the peak voltage.

(b) Peak voltage is the absolute value of the maximum deviation from 0V. If the peak voltage of a 1V RMS sine wave AC signal is measured, for example on an oscilloscope, then the measurement would be $\sqrt{2}$V or about 1.41V from zero to peak voltage.

(c) Peak-to-peak voltage is the maximum positive deviation from 0V minus the maximum negative deviation from 0V, If the peak-to-peak voltage of a 1V RMS sine wave AC signal is measured, for example on an oscilloscope, then the measurement would be $2\sqrt{2}$V or about 2.82V from negative peak to positive peak voltage.

6. In the “resonant frequency of a series RLC circuit” questions, ignore the resistor because it only affects the Q of the circuit and not the resonant frequency.
7. In the “phase angle between the voltage across and the current through a series R-L-C circuit” questions, add up the resistor values (usually only one) to get a resistance value (R) and add up the signed (inductors (XL) are positive and capacitors (XC) are negative) reactances to get a reactance value (X). Compute $\frac{X}{R}$. Noting the absolute value of the fraction, in the Extra question pools you will get one of two values: $\frac{1}{4} \Rightarrow 14.0°$ or $\frac{3}{4} \Rightarrow 36.9°$. If the sign of X is positive (inductive), then the voltage leads the current; otherwise, it is capacitive and the voltage lags the current.
8. The voltages around a complete path in a circuit adds to 0. This is the simple statement of Kirchoff’s Voltage Law.
9. The directed currents through a point in a circuit adds to 0. This is the simple statement of Kirchoff’s Current Law.
10. How to do complex multiplication and division on a calculator that only has rectangular to polar and polar to rectangular coordinate conversion.

$$(3 + 4\jmath) \times (5 - 12\jmath)$$

Convert the rectangular forms to polar forms.

$$(3 + 4\jmath) \Rightarrow 5\angle 53.13°$$

$$(5 - 12\jmath) \Rightarrow 13\angle - 67.38°$$

Compute the polar form of the product by multiplying the magnitudes and adding the angles.

$$65\angle - 14.25°$$

Convert the polar form to rectangular form.

$$63 - 16\jmath$$

To do complex division, compute the complex reciprocal of the demoninator (the lower number) and then multiply the numerator (the upper number) by the complex reciprocal.

$$\frac{63 - 16\jmath}{3 + 4\jmath}$$

$$63 - 16\jmath \Rightarrow 65\angle - 14.25°$$

$$(3 + 4\jmath) \Rightarrow 5\angle 53.13°$$

The reciprocal of $(5, 53.13°)$ has a magnitude that is the reciprocal of the original magnitude and the negative of the original angle.

$$\frac{1}{5}\angle - 53.13°$$

$$65\angle - 14.25° \times \frac{1}{5}\angle - 53.13° \Rightarrow$$

$$13\angle - 67.38° \Rightarrow 5 - 12\jmath$$

11. The input-output relationships of different type of two input logic gate:

input 1	0	0	1	1
input 2	0	1	0	1
ZERO	0	0	0	0
AND	0	0	0	1
Not I1 implies I2	0	0	1	0
I1	0	0	1	1
Not I2 implies I1	0	1	0	0
I2	0	1	0	1
XOR or not equals	0	1	1	0
OR	0	1	1	1
NOR	1	0	0	0
NXOR or equals	1	0	0	1
Not I2	1	0	1	0
I2 implies I1	1	0	1	1
Not I1	1	1	0	0
I1 implies I2	1	1	0	1
NAND	1	1	1	0
ONE	1	1	1	1

Smith Chart: 3^{rd} level description

The Smith Chart is a conformal remapping of the complex impedance or admittance (the reciprocal of impedance) plane.
A conformal mapping is one that preserves the angles present in the original mapping. In the case of the Smith Chart, the vertical lines of constant resistance of the original half plane of positive resistance become circles all tangent at positive infinite reistance and the horizonal lines of constant reactance become circular arcs which continue to intersect at right $90°$ angles. The exterior of the outer circle is the negative resistance region.

The negative resistance region is normally not present as negative resistance devices and circuits are not commonly used.
The Wikipedia article is useful.

Complex Arithmetic

Common usages in electronics

Complex arithmetic, among other uses in electronics, is used to to compute phase angles and to compute alternating-current impedances. In the Extra 2008 examination, it is sufficient for almost all questions merely to recognize the resistive and reactive components.

Representation

In rectangular coordinates:

$$a + b\jmath$$

where a is the real or resistive component and b is the imaginary or reactive component. The $\jmath$ is marker of the imaginary or reactive component and is defined as:

$$\jmath^2 = -1$$

In polar coordinates:

$$a\angle\theta$$

where a is the *how far* and θ is the *which direction*.
Conversion from rectangular to polar coordinates:

$$a + b\jmath$$

becomes:

$$\sqrt{a^2 + b^2}\angle \arctan(\frac{b}{a})$$

N.B., the $\arctan$ or $\tan^{-1}$ function on scientific calculators is what you have to use; but is not exactly what you want. If your calculator has *rectangular to polar*, then use that capacity; the proper $\arctan\left(\frac{b}{a}\right)$ is embedded in that capacity. If a is positive, then the $\arctan$ or $\tan^{-1}$ function gives the answer you want; otherwise, add $180°$ to the angle. The problem is that the quadrant of the angle can not be determined unless the sign of the denominator is known to the algorithm.
Conversion from polar to rectangular coordinates:

$$a\angle\theta$$

becomes:

$$a\cos(\theta) + a\sin(\theta)$$

N.B., many scientific calculators have builtin functions for the above coordinate conversions. Some mid and high-end scientific calculators have builtin complex arithmetic.

Addition and subtraction

$$(a + b\jmath) + (c + d\jmath) = (a + c) + (c + d)\jmath$$

Convert polar coordinates to rectangulars for addition. If necessary, then convert the rectangular coodinate result back to polar coordinates.

Multiplication

In rectangular coordinates:

$$(a + b\jmath)(c + d\jmath) = (ac - db) + (ad + cb)\jmath$$

In polar coordinates:

$$(a\angle\theta)(b\angle\phi) = ab\angle(\theta + \phi)$$

Reciprocal

In rectangular coodinates:

$$\frac{1}{a + b\jmath} = \frac{1}{a + b\jmath} \times \frac{a - b\jmath}{a - b\jmath} = \frac{a - b\jmath}{a^2 + b^2}$$

N.B., the imaginary portion reversed its sign, the value is divided by the square of the magnitude and the overall effect in polar terms is the reciprocal of the magnitude and the reversal of the angle.
In polar coordinates, the computation is much simpler:

$$\frac{1}{a\angle\theta} = (\frac{1}{a})\angle(-\theta)$$

Do division by computing the reciprocal and then multiplying by it.

Square root

Convert to polar coordinates and then:

$$\sqrt{a\angle\theta} = \sqrt{a}\angle\frac{\theta}{2} \text{ or } \sqrt{a}\angle(\frac{\theta}{2} + 180° \text{ modulus } 360°)$$

Mnemonics

Power: Apple PIE

$\text{Power} = \text{Voltage} \times \text{Current}:$

$$P = IE$$

Ohm's Law: EIRE / canal

In resistance form: $\text{Voltage} = \text{Current} \times \text{Resistance}$ or:

$$E = IR$$

In conductance form:
$\text{Voltage} \times \text{Conductance} = \text{Current}$ or:

$$ES = I$$

.

ELI the ICE man

Voltage (E) on an inductor (L) leads current (I).
Current (I) on an capacitor (C) leads voltage (E).
For a pure inductor, the phase angle is $90°$.
For a pure capacitor, the phase angle is $-90°$.

Very short trigonometric tables

These are the only angles I noticed being used in the Extra 2002 test. Pay close attention to the angles for $\arctan$ equal to 0.25, 0.50, 0.75 and the $\sin$ and $\cos$ values for $0°$, $30°$, $45°$, $60°$ and $90°$.

$\angle°$	sin	cos	tan
0.0	0.00	1.00	0.00
14.0	0.24	0.97	0.25
26.5	0.45	0.90	0.50
30.0	0.50	0.87	0.58
36.9	0.60	0.80	0.75
45.0	0.71	0.71	1.00
60.0	0.87	0.50	1.73
90.0	1.00	0.00	∞

Very small e^{-t} table

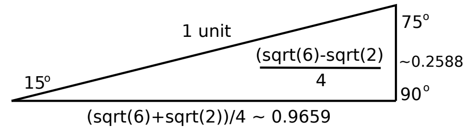
$-t$	to do	done
0	1.000	0.000
1	0.368	0.632
2	0.135	0.865
3	0.050	0.950
4	0.018	0.982
5	0.007	0.993

Metric prefix table

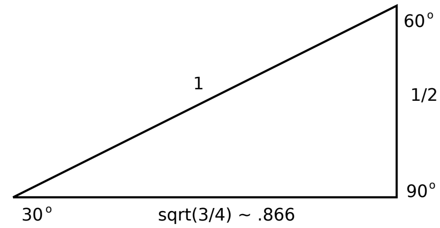
symbol	name	value
E	exa	10^{18}
P	peta	10^{15}
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
		10^{0}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}
a	atto	10^{-18}

Trigonometry

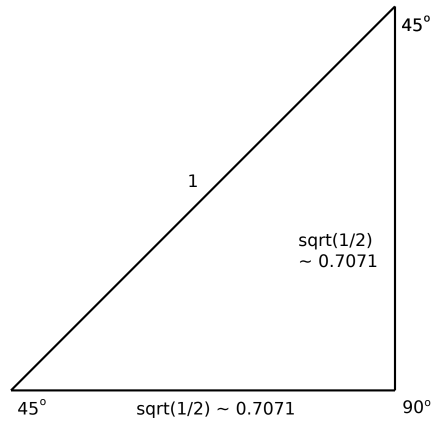
In the Extra examination, an angle accuracy of $15°$ is sufficient to recognize the correct answer.



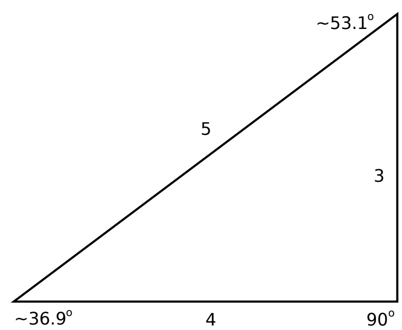
Note the approximately 4 : 1 run-to-rise ratio. $\arctan(\frac{1}{4})$ is approximately $14°$.



Note the 2 : 1 hypotenuse-to-rise. $\arcsin(\frac{1}{2})$ is $30°$.



Note the 1: 1 run-to-rise ratio. $\arctan(1)$ is $45°$.
For a $60°$ triangle, look at the $30°$ triangle sitting on its short side instead. Similarly for a $75°$ triangle, look at the $15°$ triangle sitting on its short side.



This is the simplest integer-sided right triangle. It was known to the ancient Egyptians and Chinese about 5,000 years ago. $\arctan(\frac{3}{4})$ is approximately $36.9°$. $\arctan$ is sometimes called $\tan^{-1}$ or inverse $\tan$.

Impedance and Admittance

	ohms	siemens
Real	Resistance	Conductance
Reactive	Reactance	Susceptance
Complex	Impedance	Admittance

The *ohms* and the *siemens* columns are reciprocals of each other.
Inductance is plotted in the positive reactance direction.
Capacitance is plotted in the negative reactance direction.
These directions reverse if susceptance is used.

Kirchhoff's Laws

Kirchhoff's Current Law

Kirchhoff's current law derives from the conservation of electrical charge. At any point in an electrical circuit where charge density is not changing in time, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point. A charge density changing in time would mean the accumulation of a net charge, e.g., the charging of a capacitor. If this displacement current is accounted for, then this law applies even in this case.

Kirchhoff's Voltage Law

Kirchhoff's voltage law derives from the conservation of energy. The directed voltage drops around a closed loop in a circuit, in the absence of changing magnetic fields is zero. If the induced voltages from changing magnetic fields are accounted for, then this law applies in full. A power supply opposes the passive voltages in the circuit.

Reactance and Frequency formulae

$$X_L = 2\pi f L \jmath$$

Inductance increases when frequency increases.

$$X_C = \frac{-\jmath}{2\pi f C}$$

Capacitance decreases when frequency increases.
The 2π is a conversion factor from radians per second to cycles per second (Hertz or Hz). L is in Henrys. C is in Farads. X_L and X_C are in Ohms.
Resonance occurs when $X_L = -X_C$.

$$2\pi f L \jmath = -\frac{-\jmath}{2\pi f C}$$

$$(2\pi f L)(2\pi f C) = 1$$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

N.B., in a series resonant circuit, the reactances cancel and the remaining resistance is low; in a parallel resonant circuit, the susceptances cancel and the remaining resistance is high.

Computation Rules

1. Resistors and inductors are "voltage" devices.
2. Capacitors are "current" devices.
3. Series voltages add: do these computations in impedance (ohms, Ω).
4. Parallel currents add: do these computations in admittance (siemans, S).
5. The parallel resistor formula derives directly from this.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$S_1 = \frac{1}{R_1}, S_2 = \frac{1}{R_2}$$

$$S_T = S_1 + S_2$$

$$R_T = \frac{1}{S_T}$$

Q and $\frac{1}{2}$ power bandwidth

At resonance, the inductive and capacitive reactances are equal in magnitude and opposite in sign. In the equations below, X is the magnitude of either the inductive and capacitive reactance.

For series resonant circuits, using ohms:

$$Q = \frac{X}{R}$$

For parallel resonant circuits, using ohms:

$$Q = \frac{R}{X}$$

For parallel resonant circuits, using suspectances in siemens:

$$Q = \frac{X}{R}$$

The $\frac{1}{2}$ power bandwidth, W, of a resonant circuit:

$$W = \frac{f}{Q}$$

At resonance, series RLC resonant circuits have minimum resistance without any reactance.

At resonance, parallel RLC resonant circuits have minimum conductance without any suspectance.

RL and RC circuits

These circuits are generally used for timing or filtering and are fed a DC step function from a switch being changed or a change in a digital logic signal.

One time constant is RL or RC.

The fraction of the building of the magnetic field in the inductor or the electrostatic field in the capacitor is

$1 - e^{-t}$. Normally, after 5 time constants have passed the charging is considered complete.

In the questions, pay very close attention to whether *how much has been done* $(1 - e^{-t})$ or *how much remains to be done* (e^{-t}) is asked.

You do not have to know calculus and differential equations to answer the Extra class test questions; but, it can help.

e^x is both its own derivative and its own integral. Those facts come into play as the charging rate for a capacitor (it charges by storing electrical charge) or inductor (it charges by storing magnetic field) is proportional to amount left to be charged.

$$\int e^x = e^x + C$$

$$\frac{de^x}{dx} = e^x$$

In particular, the solution of the differential equation with an initial voltage

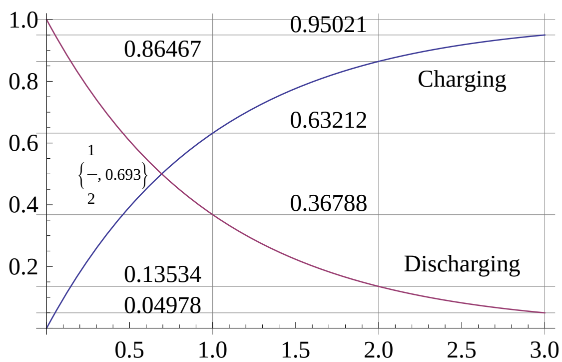
$$\frac{dv(t)}{dt} = v_{\text{final}} - v(t)$$

$$v(0) = v_{\text{initial}}$$

is:

$$v(t) = e^{-t} \times ((e^t - 1) \times v_{\text{final}} + v_{\text{initial}}$$

For a charging capacitor, with $\text{voltage}_{\text{initial}} = 0$ and $v_{\text{final}} = 1$, the solution simplifies to $1 - e^{-t}$. For a discharging capacitor, with $v_{\text{initial}} = 1$ and $v_{\text{final}} = 0$, the solution simplifies to e^{-t}.



The only signifance of the intersection of the two graphs is that when the job is half done at $t \approx 0.693$.

This is the reason that e^x and the inverse function to e^x, $\ln(x)$ occurs in RL and RC problems.

Power factor

The power factor of a circuit is $\cos(\text{ phase angle })$.

A few worked problems

E5C11

What do the two numbers represent that are used to define a point on a graph using rectangular coordinates?

The coordinate values along the horizontal and vertical axes

The key is "rectangular".

These axes are sometimes called the x (horizontal) axis and y (vertical) axis. The alternate, commonly used coordinate system is the polar coordinate system with r (distance or magnitude) and θ (angle or phase).

E4B15

Which of the following can be used as a relative measurement of the Q for a series-tuned circuit?

The bandwidth of the circuit's frequency response.

A narrow bandwidth indicates a high Q and a wide bandwith indicates a low Q.

A crystal has a very high Q, which makes it very stable, which means its frequency varies over a very narrow range, which means it has a very narrow bandwidth.

E5A01

What can cause the voltage across reactances in series to be larger than the voltage applied to them?

Resonance.

It is the "pumping" action of the driving voltage at the resonant frequency that causes the energy stored in the resonant circuit to build up.

Compare this to pumping on a playground swing: a series of relatively mild pumping actions build up to a very large magnitude swinging motion.

E5A02

What is resonance in an electrical circuit?

The frequency at which the magnitude of capacitive reactance, which is negative, equals the magnitude of the inductive reactance is the resonance frequency.

The "correct" answer does *not* mention *magnitude*.

In a parallel resonant circuit, the combined reactance is resistive and very high, depending on Q, which is dependent on the resistance in the circuit.

$$\frac{1}{\frac{1}{\jmath} + \frac{1}{-\jmath}}$$

$$\frac{1}{\frac{-\jmath}{1} + \frac{\jmath}{1}}$$

$$\frac{1}{-\jmath + \jmath}$$

$$\frac{1}{0}$$

∞

In a series resonant circuit, the combined reactance is resistive and very low, depending on Q, which is dependent on the resistance in the circuit.

$\jmath - \jmath$

0

E5A03

(D) What is the magnitude of the impedance of a series RLC circuit at resonance?

Approximately equal to circuit resistance.

See the discussion of combined reactance in E5A02 above.

E5A04

What is the magnitude of the impedance of a circuit with a resistor, an inductor and a capacitor all in parallel, at resonance?

Approximately equal to circuit resistance.

See the discussion of combined reactance in E5A02 above.

E5A05

What is the magnitude of the current at the input of a series RLC circuit as the frequency goes through resonance?

Maximum.

This is an application of Ohm's Law ($\frac{E}{R} = I$. As the reactance goes down as resonance is approached, under the same supply voltage, the current increases

E5A06

What is the magnitude of the circulating current within the components of a parallel LC circuit at resonance?

It is at a maximum.

The parallel inductor and capacitor are *effectively* in series with each other. The reactance goes towards zero and the circuilating current maximizes. But, note well the next question (E5A07) which is about the input current.

The distracting formulae are for resonant frequency and reactance of the inductor, which are not needed here.

E5A07

What is the magnitude of the current at the input of a parallel RLC circuit at resonance?

Minimum.

Note well the difference in wording of this question and of E5A06. This question is about input current and E5A06 is about circulating current.

See the discussion of combined reactance in E5A02 above.

E5A08

What is the phase relationship between the current through and the voltage across a series resonant circuit at resonance?

The voltage and current are in phase.

At resonance, the reactive components have canceled and the circuit is resistive. That, is, the voltage and current are in phase.

Fortunately, if you mistake this question for the next question, the answer is the same!

E5A09

What is the phase relationship between the current through and the voltage across a parallel resonant circuit at resonance?

The voltage and current are in phase.

At resonance, the reactive components have canceled and the circuit is resistive. That, is, the voltage and current are in phase.

Fortunately, if you mistake this question for the previous question, the answer is the same!

E5A10

What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 1.8 MHz and a Q of 95?

18.9kHz.

1.8 MHz divided by 95 giving 18947 Hz or 18.9kHz.

E5A11

What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 7.1 MHz and a Q of 150?

47.3 kHz.

7.1MHz divided by 150 giving 47333 Hz or 47.3kHz.

E5A12

What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 3.7 MHz and a Q of 118?

31.4 kHz.

3.7 MHz divided by 118 giving 31355.9Hz or 31.4kHz.

E5A13

What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 14.25 MHz and a Q of 187?

76.2 kHz.

14.25 MHz divided by 187 giving 76203Hz or 76.2kHz.

E5A14

What is the resonant frequency of a series RLC circuit if R is 22 ohms, L is 50 microhenrys and C is 40 picofarads?

3.56MHz.

First, ignore the resistor — it affects the Q but not the resonant frequency!

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 40 \times 10^{-12}}}$$

$$f = \frac{1}{2\pi\sqrt{2000 \times 10^{-18}}}$$

$$f = \frac{1}{2 \times 3.1416 \times 44.72 \times 10^{-9}}$$

$$f = \frac{10^9}{281}$$

$$f = 3.56 \text{ MHz}$$

E5A15

What is the resonant frequency of a series RLC circuit if R is 56 ohms, L is 40 microhenrys and C is 200 picofarads?

1.78MHz.

First, ignore the resistor — it affects the Q but not the resonant frequency!

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{40 \times 10^{-6} \times 200 \times 10^{-12}}}$$

$$f = \frac{1}{2\pi\sqrt{8000 \times 10^{-18}}}$$

$$f = \frac{1}{2 \times 3.1416 \times 89.44 \times 10^{-9}}$$

$$f = \frac{10^9}{562}$$

$$f = 1.78 \text{ MHz}$$

E5A16

What is the resonant frequency of a parallel RLC circuit if R is 33 ohms, L is 50 microhenrys and C is 10 picofarads?

7.12 MHz.

First, ignore the resistor — it affects the Q but not the resonant frequency!

Second, pay attention to your units and prefixes. The distractors differs in their unit prefixes!

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 10 \times 10^{-12}}}$$

$$f = \frac{1}{2\pi\sqrt{500 \times 10^{-18}}}$$

$$f = \frac{1}{2 \times 3.1416 \times 22.36 \times 10^{-9}}$$

$$f = \frac{10^9}{281}$$

$$f = 7.1 \text{ MHz}$$

E5A17 (A) What is the resonant frequency of a parallel RLC circuit if R is 47 ohms, L is 25 microhenrys and C is 10 picofarads? A. 10.1 MHz

First, ignore the resistor — it affects the Q but not the resonant frequency!

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{25 \times 10^{-6} \times 00 \times 10^{-12}}}$$

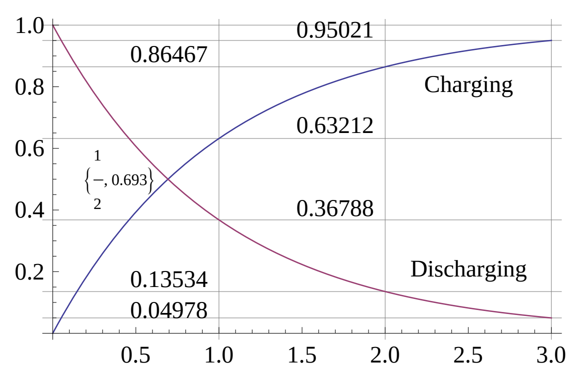
$$f = \frac{1}{2\pi\sqrt{250 \times 10^{-18}}}$$

$$f = \frac{1}{2 \times 3.1416 \times 15.81 \times 10^{-9}}$$

$$f = \frac{10^9}{99.346}$$

$$f = 10.1 \text{ MHz}$$

RC or RL, 3 Time Constants



E5B01

What is the term for the time required for the capacitor in an RC circuit to be charged to 63.2% of the supply voltage?

One time constant.

How much charging remains as a fraction: 0.368.

$$\ln(0.368) = -1$$

The answer is one time constant. The time interval always is measured in "time constants."

E5B02

What is the term for the time it takes for a charged capacitor in an RC circuit to discharge to 36.8

One time constant

See discussion in E5B01!

E5B03

The capacitor in an RC circuit is discharged to what percentage of the starting voltage after two time constants?

How much charge remains after two time constants? 13.5%.

$$e^{-2} \approx 0.135$$

.

E5B04

What is the time constant of a circuit having two 220-microfarad capacitors and two 1-megohm resistors, all in parallel? 220 seconds.

2 parallel 220μF capacitors is 440μF. 2 parallel 1MΩ is 0.5MΩ.

$$440 \times 10^{-6} \times 0.5 \times 10^{6} = 220$$

The micro and meg factors cancel to 1.

E5B05

How long does it take for an initial charge of 20 V DC to decrease to 7.36 V DC in a 0.01-microfarad capacitor when a 2-megohm resistor is connected across it?

0.02 seconds.

$$\ln(7.36/20) = \ln(.368) \approx -1$$

One time constant.

$$0.01\mu F \times 2M\Omega = 0.02 \text{ seconds}$$

E5B06

How long does it take for an initial charge of 800 V DC to decrease to 294 V DC in a 450-microfarad capacitor when a 1-megohm resistor is connected across it?

450 seconds.

$$\ln(294/800) = \ln(.3675) \approx -1$$

$$450\mu F \times 1M\Omega = 450 \times 1 = 450 \text{ seconds}$$

Watch your units and prefixes!

E5B07

What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 500 ohms, R is 1 kilohm, and XL is 250 ohms?

14.0 degrees with the voltage lagging the current.

Reactance is $-500\jmath\Omega + 250\jmath\Omega = -250\jmath\Omega$. Restance is 1000Ω. This a 1-to-4 rise-over-run triangle giving $14°$. Now, remember Eli the ice man: the current leads the voltage or the voltage lags the current.

Remember those four triangles!

E5B08

What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 100 ohms, R is 100 ohms, and XL is 75 ohms?

14 degrees with the voltage lagging the current

Reactance is $-100\jmath\Omega + 75\jmath\Omega = -25\jmath\Omega$. Restance is 100Ω. This a 1-to-4 rise-over-run triangle giving $14°$. Now, remember Eli the ice man: the current leads the voltage or the voltage lags the current.

Remember those four triangles!

E5B09

What is the relationship between the current through a capacitor and the voltage across a capacitor?

Current leads voltage by 90 degrees.

Remember Eli the ice man: the current leads the voltage or the voltage lags the current.

E5B10

What is the relationship between the current through an inductor and the voltage across an inductor?

Voltage leads current by 90 degrees.

Remember Eli the ice man: the voltage leads the current or the current lags the voltage.

E5B11

What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 25 ohms, R is 100 ohms, and XL is 50 ohms?

14 degrees with the voltage leading the current

Reactance is $-25\jmath\Omega + 50\jmath\Omega = 25\jmath\Omega$. Restance is 100Ω. This a 1-to-4 rise-over-run triangle giving $14°$. Now, remember Eli the ice man: the current lags the voltage or the voltage leads the current.

Remember those four triangles!

E5B12

What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 75 ohms, R is 100 ohms, and XL is 50 ohms?

14 degrees with the voltage lagging the current.

Reactance is $-100\jmath\Omega + 75\jmath\Omega = -25\jmath\Omega$. Restance is 100Ω. This a 1-to-4 rise-over-run triangle giving $14°$. Now, remember Eli the ice man: the current leads the voltage or the voltage lags the current.

Remember those four triangles!

E5B13

What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 250 ohms, R is 1 kilohm, and XL is 500 ohms?

14.04 degrees with the voltage leading the current.

Reactance is $-250\jmath\Omega + 500\jmath\Omega = 25\jmath\Omega$. Restance is 1000Ω. This a 1-to-4 rise-over-run triangle giving $14°$. Now, remember Eli the ice man: the current lags the voltage or the voltage leads the current.

Remember those four triangles!

E5C01

In polar coordinates, what is the impedance of a network consisting of a 100-ohm-reactance inductor in series with a 100-ohm resistor?

141 ohms at an angle of 45 degrees.

This is a run equals rise triangle. Remember Eli the ice man: the voltage leads the current (the angle is positive).

The hypotenuse is $\sqrt{2} \approx 1.41$ times the length of the sides and the angle is $45°$.

Remember those four triangles!

E5C02

In polar coordinates, what is the impedance of a network consisting of a 100-ohm-reactance inductor, a 100-ohm-reactance capacitor, and a 100-ohm resistor all connected in series?

100 ohms at an angle of 0 degrees.

The impedance of the inductor and the capacitor cancel. Therefore, the answer is $100\angle 0°$ or 100 ohms pure resistive.

E5C03

In polar coordinates, what is the impedance of a network consisting of a 300-ohm-reactance capacitor, a 600-ohm reactance inductor, and a 400-ohm resistor, all connected in series?

500 ohms at an angle of 37 degrees.

This is a 3-4-5 triangle with the smallest angle being $\approx 36.9°$. Remember Eli the ice man: the voltage leads the current (the angle is positive).

Remember those four triangles!

E5C04

In polar coordinates, what is the impedance of a network consisting of a 400-ohm-reactance capacitor in series with a 300-ohm resistor?

500 ohms at an angle of -53.1 degrees

This is a 3-4-5 triangle with the larger acute angle being $\approx 53.1°$ or $90° - 36.9°$. Remember Eli the ice man: the voltage lags the current (the angle is negative).

Remember those four triangles!

E5C05

In polar coordinates, what is the impedance of a network consisting of a 400-ohm-reactance inductor in parallel with a 300-ohm resistor?

240 ohms at an angle of 36.9 degrees.

$$\frac{1}{\frac{1}{300} + \frac{1}{400\jmath}}$$

$$\frac{1}{\frac{4}{1200} + \frac{-3\jmath}{1200}}$$

$$\frac{1}{\frac{4-3\jmath}{1200}}$$

$$\frac{1200}{4-3\jmath}$$

$$\frac{1200}{5\angle -36.9°}$$

$$240\angle 36.9°$$

Note that $\frac{1}{i} = -i$. In the rectangular to polar conversion, the 3-4-5 triangle was used.

Remember those four triangles!

E5C06

In polar coordinates, what is the impedance of a network consisting of a 100-ohm-reactance capacitor in series with a 100-ohm resistor?

141 ohms at an angle of -45 degrees

This is a run equals rise triangle. Remember Eli the ice man: the current leads the voltage (the angle is negative).

The hypotenuse is $\sqrt{2} \approx 1.41$ times the length of the sides and the angle is $45°$.

Remember those four triangles!

E5C07

In polar coordinates, what is the impedance of a network comprised of a 100-ohm-reactance capacitor in parallel with a 100-ohm resistor?

71 ohms at an angle of -45 degrees

$$\frac{1}{\frac{1}{100} + \frac{1}{-100\jmath}}$$

$$\frac{1}{\frac{1}{100} + \frac{\jmath}{100}}$$

$$\frac{1}{\frac{1-\jmath}{100}}$$

$$\frac{100}{1+\jmath}$$

$$\frac{100}{\sqrt{2}\angle 45^\circ}$$

$$\approx 71\angle-45^\circ$$

Note that $\frac{1}{i} = -i$. In the rectangular to polar conversion, the run equals rise triangle was used.

Remember those four triangles!

E5C08

In polar coordinates, what is the impedance of a network comprised of a 300-ohm-reactance inductor in series with a 400-ohm resistor?

500 ohms at an angle of 37 degrees.

This is the 3-4-5 triangle. The hypotenuse is 5 units. The unit is 100Ω. The angle is $\approx 36.9^\circ$.

Remember those four triangles!

E5C09

When using rectangular coordinates to graph the impedance of a circuit, what does the horizontal axis represent?

Resistive component.

This is the convention. You just have to memorize this answer.

E5C10

When using rectangular coordinates to graph the impedance of a circuit, what does the vertical axis represent?

Reactive component.

This is the convention. You just have to memorize this answer.

E5C11

What do the two numbers represent that are used to define a point on a graph using rectangular coordinates?

The coordinate values along the horizontal and vertical axes.

The key is "rectangular". Polar corrdinates use magnitude and angle.

E5C12

If you plot the impedance of a circuit using the rectangular coordinate system and find the impedance point falls on the right side of the graph on the horizontal axis, what do you know about the circuit?

It is equivalent to a pure resistance.

This is the convention. You just have to memorize this answer.

E5C13

What coordinate system is often used to display the resistive, inductive, and/or capacitive reactance components of an impedance?

Rectangular coordinates.

This is the convention. You just have to memorize this answer.

E5C14

What coordinate system is often used to display the phase angle of a circuit containing resistance, inductive and/or capacitive reactance?

Polar coordinates.

This is the convention. You just have to memorize this answer. The mention of "phase angle" indicates a polar coordinate system.

E5C15

In polar coordinates, what is the impedance of a circuit of 100 -j100 ohms impedance?

141 ohms at an angle of -45 degrees.

This is a run equals rise triangle. The hypotenuse is $\sqrt{2}$ times the length of the sides.

Remember Eli the ice man: the current leads the voltage (the angle is negative).

Remember those four triangles!

E5C16

In polar coordinates, what is the impedance of a circuit that has an admittance of 7.09 millisiemens at 45 degrees?

141 ohms at an angle of -45 degrees

$$\frac{1}{7.09\times10^{-3}} \Rightarrow 141\Omega\angle-45^\circ$$

E5C17

In rectangular coordinates, what is the impedance of a circuit that has an admittance of 5 millisiemens at -30 degrees?

173 + j100 ohms.

The impedance is requested. Therefore, the admittance needs to be converted to an impedance by computing the complex reciprocal in polar form. The impedance is 200 ohms at +30 degrees. This is the 30° triangle with sides of 1 : $\frac{1}{2}$: $\sqrt{\frac{3}{4}}$ or about 1 : 0.5 : 0.866.

The hypotenuse is $200\Omega\angle 30^\circ$. The resistive side is about 0.866 of the hypotenuse or 173Ω. The reactive side is positive or inductive and is 0.5 of the hypotenuse or $100\jmath\Omega$.

Therefore, the answer is $173 + 100\jmath\,\Omega$.

Remember Eli the ice man: the current leads the voltage (the angle is negative).

Remember those four triangles!

E5C18

In polar coordinates, what is the impedance of a series circuit consisting of a resistance of 4 ohms, an inductive reactance of 4 ohms, and a capacitive reactance of 1 ohm?

5 ohms at an angle of 37 degrees

This is a 3-4-5 triangle. The capacitor cancels 1 unit of the inductor's reactance.

Remember Eli the ice man: the current leads the voltage (the angle is negative).

Remember those four triangles!

E5C19

Which point on Figure E5-2 best represents that impedance of a series circuit consisting of a 400 ohm resistor and a 38 picofarad capacitor at 14 MHz?

Point 4.

If you trust them to give a correct answer, then this one is easy. The capacitor has a negative reactance. The resistor is 400Ω. QED

E5C20

Which point in Figure E5-2 best represents the impedance of a series circuit consisting of a 300 ohm resistor and an 18 microhenry inductor at 3.505 MHz?

Point 3.

If you trust them to give a correct answer, then this one is easy. The inductor has a positive reactance. The resistor is 300Ω. QED

E5C21

Which point on Figure E5-2 best represents the impedance of a series circuit consisting of a 300 ohm resistor and a 19 picofarad capacitor at 21.200 MHz?

Point 1.

If you trust them to give a correct answer, then this one is easy. The capacitor has a negative reactance. The resistor is 300Ω. QED

E5C22

In rectangular coordinates, what is the impedance of a network consisting of a 10-microhenry inductor in series with a 40-ohm resistor at 500 MHz?

40 + j31,400.

If you trust them to give a correct answer, then this one is easy. The inductor has a positive reactance. The resistor is 40Ω. QED

E5C23

Which point on Figure E5-2 best represents the impedance of a series circuit consisting of a 300-ohm resistor, a 0.64-microhenry inductor and an 85-picofarad capacitor at 24.900 MHz?

Point 8

If you trust them to give a correct answer, then this one is easy. The inductor (640pH) has a significantly greater magnitude positive reactance than the capacitor (85pF) has a negative reactance magnitude. The resistor is 300Ω. QED

E5D01

What is the result of skin effect?

As frequency increases, RF current flows in a thinner layer of the conductor, closer to the surface.

E5D02

Why is the resistance of a conductor different for RF currents than for direct currents?

Because of skin effect

E5D03

What device is used to store electrical energy in an electrostatic field?

A capacitor

The key is "static".

E5D04

What unit measures electrical energy stored in an electrostatic field?

Joule.

"If it is energy, then it is measured in Joules or an equivalent."

A Watt is the delivery of a Joule per second. A Coulomb is the result of the delivery of an Ampere for a second. A Volt is " the difference in electric potential across a wire when an electric current of one ampere dissipates one watt of power."

E5D05

Which of the following creates a magnetic field?

Electric current.

E5D06

In what direction is the magnetic field oriented about a conductor in relation to the direction of electron flow?

In a direction determined by the left-hand rule.

The generated magnetic field is perpendicular to the current. Only one of the answer is perpendicular to the current or flow. The left-hand rule applies with the electron flow is in the direction of the thumb. The left-hand rule applies with current in the direction of the thumb. The problem comes from the fact that Benjamin Franklin got the sign of the charge of the electron as negative.

E5D07

What determines the strength of a magnetic field around a conductor?

The amount of current.

E5D08

What type of energy is stored in an electromagnetic or electrostatic field?

Potential energy.

“Stored” is “potential.”

E5D09

What happens to reactive power in an AC circuit that has both ideal inductors and ideal capacitors?

It is repeatedly exchanged between the associated magnetic and electric fields, but is not dissipated

The key is “ideal.”

E5D10

How can the true power be determined in an AC circuit where the voltage and current are out of phase?

By multiplying the apparent power times the power factor.

E5D11

What is the power factor of an R-L circuit having a 60 degree phase angle between the voltage and the current?

0.5

This is a $\frac{1}{2}$ rise over run triangle.

Remember those four triangles!

E5D12

How many watts are consumed in a circuit having a power factor of 0.2 if the input is 100-V AC at 4 amperes?

80 watts.

$$P = I \times E \times pf$$

$$P = 100 \times 4 \times 0.2$$

$$P = 80$$

E5D13

How much power is consumed in a circuit consisting of a 100 ohm resistor in series with a 100 ohm inductive reactance drawing 1 ampere?

100 Watts.

$$P = I^2R$$

$$P = 1^2 100$$

$$P = 100$$

E5D14

What is reactive power?

Wattless, nonproductive power.

E5D15

What is the power factor of an RL circuit having a 45 degree phase angle between the voltage and the current?

0.707

This is a rise equals run triangle. If the hypotenuse is 1, then the sides are $\frac{1}{\sqrt{2}}$.

Remember those four triangles!

E5D16

What is the power factor of an RL circuit having a 30 degree phase angle between the voltage and the current?

0.866

This is a $\frac{1}{2}$ rise over run triangle.

Remember those four triangles!

E5D17

How many watts are consumed in a circuit having a power factor of 0.6 if the input is 200V AC at 5 amperes?

600 watts.

$$P = I \times E \times pf$$

$$P = 200 \times 5 \times 0.6$$

$$P = 600$$

E5D18

How many watts are consumed in a circuit having a power factor of 0.71 if the apparent power is 500 VA?

355 W.

$$0.71 \times 500$$

E6D0

What core material property determines the inductance of a toroidal inductor?

Permeability.

E6D07

What is the usable frequency range of inductors that use toroidal cores, assuming a correct selection of core material for the frequency being used?

From less than 20 Hz to approximately 300 MHz.

E6D08

What is one important reason for using powdered-iron toroids rather than ferrite toroids in an inductor?

Powdered-iron toroids generally maintain their characteristics at higher currents.

E6D09

What devices are commonly used as VHF and UHF parasitic suppressors at the input and output terminals of transistorized HF amplifiers?

Ferrite beads.

E6D10

What is a primary advantage of using a toroidal core instead of a solenoidal core in an inductor?

Toroidal cores confine most of the magnetic field within the core material.

This significant reduces coupling between nearby inductors.

E6D11

How many turns will be required to produce a 1-mH inductor using a ferrite toroidal core that has an inductance index (A L) value of 523 millihenrys/1000 turns?

43 turns.

$$L = A_L(\frac{N}{per-turns})^2$$

$$1 = 523(\frac{N}{1000})^2$$

$$1000000 = 523N^2$$

$$\frac{1000000}{523} = N^2$$

$$N = \sqrt{\frac{1000000}{523}}$$

$$N \approx 43.7$$

E6D12

How many turns will be required to produce a 5-microhenry inductor using a powdered-iron toroidal core that has an inductance index (A L) value of 40 microhenrys/100 turns?

35 turns.

$$L = A_L(\frac{N}{per-turns})^2$$

$$5 = 40(\frac{N}{100})^2$$

$$50000 = 40N^2$$

$$\frac{50000}{40} = N^2$$

$$N = \sqrt{\frac{50000}{40}}$$

$$N \approx 35.3$$

E9H01

What is the effective radiated power relative to a dipole of a repeater station with 150 watts transmitter power output, 2-dB feed line loss, 2.2-dB duplexer loss and 7-dBd antenna gain?

$$150 \times 10^{\frac{-2-2.2+7.2.8}{10}} \Rightarrow 150 \times 10^{0.28}$$

$$\Rightarrow 150 \times 1.95 \Rightarrow 286$$

The answer is 286W.

Extra 2002: E5C11

There does not seem to be any problem quite like this problem in the Extra 2008 and 2012 examination pools.

In polar coordinates, what is the impedance of a network comprised of a 100- picofarad capacitor in parallel with a 4,000-ohm resistor at 500 kHz?

It is a parallel circuit; therefore, work in siemens.

The conductance of a 4KΩ resistor is 0.00025 siemens.

The reactance of a 100pf capacitor at 500KHz is $\frac{-\jmath}{2\pi\times500\times10^3\times100\times10^{-12}}$, $\frac{-10000\jmath}{\pi}$ or about -3183$\jmath$ ohms. The susceptance is 0.00031416$\jmath$ siemens.

N.B., the sign changed from - to + because of the formula for computing the reciprocal of a complex number.

The admittance is $0.00025 + 0.00031416\jmath$ siemens.

The answer is requested in polar coordinates. If you have a calculator that does conversions between rectangular and polar coordinates, then use it to do the following conversion calculations.

$\arctan(\frac{0.00031416}{0.00025}$ is $\arctan(1.25664)$ or $51.49°$.

$\sqrt{0.00025^2 + 0.00031416^2}$ is 0.000401593.

The admittance in polar coordinates is $0.000401593\angle 51.49°$.

The reciprocal of a complex number in polar form is the reciprocal of the magnitude and a change of sign of the angle.

The impedance of the circuit in polar coordinates is $2490\angle - 51.49°$.

Mathematics beyond the Extra class test

Many problems of interest to amateur radio operators involve considerably more computation than is practical on a non-programable calculator. Software defined radios are. e.g., dependent on digital fast Fourier transforms and inverse transforms. Antenna design using the Numerical Electromagnetics Code involves the numerical solution of integral equations. Circuit analysis using SPICE uses integral, differential and matric equations.

Great circle paths

There are more accurate formulae than those I present here. These are sufficiently accurate for the purpose of pointing almost all antennae and for estimating great circle distances between stations. The assumption of a spherical Earth instead of a prolate ellipsoid with an even smaller third order deviation from that figure causes at most about a 0.3% error. If the other station is close to being antipodal from you than the aiming direction does not matter.

If you are interested in more accurate formulae, then check the Wikipeda article: "http://en.wikipedia.org/wiki/Vincenty%27s_formulae".

The data needed: the latitude and longitude of your station and the latitude and longitude of other station. You can get this from Google Earth or a GNSS receiver.

For the sake of this example:

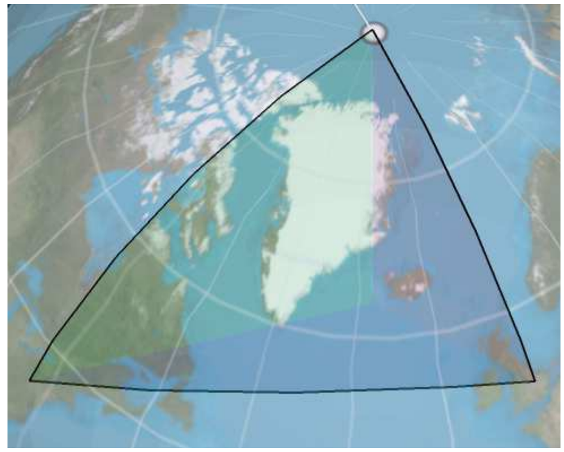
- $38.897511°N \quad 77.036551°W$ Washington, DC
- $51.5010991°N \quad 0.142472°W$ London, England

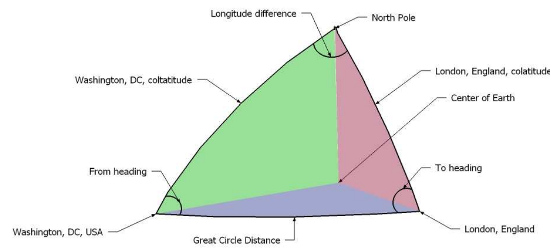
Since the locations are closer to the North Pole than the South, we will use North colatitudes (angles from the North Pole).

Since the east bound longitude difference is smaller than the west bound difference, we will use that longitude difference.

1. Washington colatitude C_F in decimal degrees $51.102489°$.
2. London colatitude C_T in decimal degrees $38.4989009°$.
3. longitude difference LD east bound in decimal degrees $76.894079°$.

These angles must all be less than 180 degrees.





We have the situation of knowing two sides (C_F and C_T) of the spherical triangle and the included angle between those sides (LD). We want the third side (D, the great circle distance) and the included angles at Washington (H_F, the from or departure heading) and at London (H_T, the to or arrival heading). D is considered an angle at the center of the spherical Earth.

$$\cos(D) = \cos(C_F) \times \cos(C_T) +$$

$$\sin(C_F) \times \sin(C_T) \times \cos(\text{ LD })$$

$$\cos(D) = 0.6279292492 \times 0.7826200984 +$$

$$0.7782704273 \times 0.6224996238 \times 0.2267519578$$

$$\cos(D) = 0.601285263$$

$$D = 53.0379966085°$$

The average circumference of the Earth is 40.0007516 Mm or 24,901.55 mi.

W1AW $\frac{53.0379966085}{360} \times 24,901.55 \approx 3668.69$ mi .

The sine rule for spherical triangles is:

$$\frac{\sin(C_F)}{\sin(H_T)} = \frac{\sin(C_T)}{\sin(H_F)} = \frac{\sin(D)}{\sin(\text{ LD })}$$

or:

$$\frac{\sin(H_T)}{\sin(C_F)} = \frac{\sin(H_F)}{\sin(C_T)} = \frac{\sin(\text{ LD })}{\sin(D)}$$

Giving:

$$H_F = \arcsin(\sin(C_T) \times \frac{\sin(\text{ LD })}{\sin(D)})$$

and

$$H_T = \arcsin(\sin(C_F) \times \frac{\sin(\text{ LD })}{\sin(D)})$$

The Washington departure angle is:

$$\arcsin(\sin(38.4989009) \times \frac{\sin(76.894079)}{\sin(53.0379966085)}$$

The departure heading is about $49.4°$ east of North. The London arrival angle is:

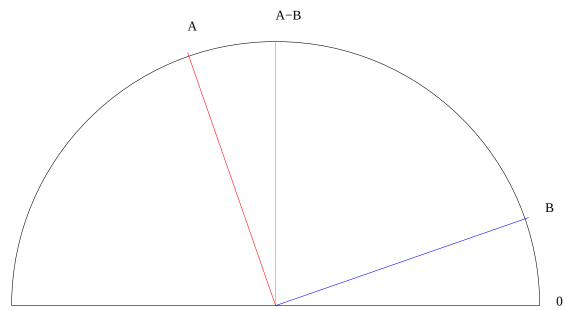
$$\arcsin(\sin(51.102489) \times \frac{\sin(76.894079)}{\sin(53.0379966085)}$$

The arrival heading heading is about $71.6°$ west of North.

A method of illustrating the departure heading and distance to another point of the globe from a given location is an Azimuthal Equidistant map generated for that given location. The outer circle represents the antipodal point. In the case of the map below, the given location is Chicago, Illinois, USA. The map was generated using Wolfram Mathematica 8.

```
<< WorldPlot‘

AzimuthalEqualDistance[ilat_,ilong_]
:= (
  m2r = N[Pi / 10800];
  lat = ilat m2r;
  long = ilong m2r;
  c = N[ArcCos[
        Sin[latc] Sin[lat]+
        Cos[latc]Cos[lat]
          Cos[long-longc]]];
  kk = N[c / Sin[c]];
  N[kk{Cos[lat]Sin[long-longc],
      (Cos[latc]Sin[lat]-
       Sin[latc]Cos[lat]
          Cos[long-longc])}])

latc = 41.82Degree

longc = -87.62Degree

worldmap = WorldPlot[
  {World,Function[GrayLevel[.6]]},
  WorldProjection ->
    AzimuthalEqualDistance,
  WorldPoints-> 1024,
  WorldBorders-> Thickness[0.002],
  WorldToGraphics-> True]

Show[{worldmap,
      Graphics[Circle[{0,0},N[Pi]]]}]
```

Trigonometric Sum and Difference of Angles Formulae



This material is in the 3^{rd} degree.

The clockwise angle from A to $A - B$ is the same as the angle from B to 0.

The lines from the center of the semicircle to the arc are all radii of the semicircle and therefore the same length.

By the Side-Angle-Side theorem of geometry, the distance from point 0 on the semicircle arc to the point $B-A$ is the same distance as from point B to the point A on the semicircle arc.

$$\sqrt{(\cos(A-B)-1)^2 + (\sin(A-B)-0)^2} = \sqrt{(\cos(A)-\cos(B))^2 + (\sin(A)-\sin(B))^2}$$

$$(\cos(A-B)-1)^2 + (\sin(A-B)-0)^2 = (\cos(A)-\cos(B))^2 + (\sin(A)-\sin(B))^2$$

$$\cos^2(A-B) - 2*\cos(A-B) + 1 + \sin^2(A-B) = \cos^2(A) - 2*\cos(A)*\cos(B) + \cos^2(B) + \sin^2(A) - 2*\sin(A)*\sin(B) + \sin^2(B)$$

$$\cos^2(x) + \sin^2(x) = 1$$

$$2 - 2 * \cos(A - B) =$$

$$2 - 2 * (\cos(A) * \cos(B) + \sin(A) * \sin(B))$$

$$\cos(A - B) = \cos(A) * \cos(B) + \sin(A) * \sin(B)$$

$$\cos(x) = \cos(-x), \sin(x) = -\sin(-x)$$

$$\cos(A - (-B)) =$$

$$\cos(A) * \cos(-B) + \sin(A) * \sin(-B)$$

$$\cos(A + B) = \cos(A) * \cos(B) - \sin(A) * \sin(B)$$

$$\cos(x - 90^\circ) = \sin(x), \sin(x - 90^\circ) = -\cos(x)$$

$$\cos((A + B) - 90^\circ) = \cos(A + (B - 90^\circ))$$

$$\sin(A + B) =$$

$$\cos(A) * \cos(B - 90^\circ) - \sin(A) * \sin(B - 90^\circ)$$

$$\sin(A + B) = \cos(A) * \sin(B) + \sin(A) * \cos(B)$$

$$\sin(A - B) =$$

$$\cos(A) * \sin(-B) + \sin(A) * \cos(-B)$$

$$\sin(A - B) =$$

$$-\cos(A) * \sin(B) + \sin(A) * \cos(B)$$

Here are the formulae for the $\sin$ and $\cos$ of the sum and differences of two angles:

$$\cos(A + B) = \cos(A) * \cos(B) - \sin(A) * \sin(B)$$

$$\cos(A - B) = \cos(A) * \cos(B) + \sin(A) * \sin(B)$$

$$\sin(A + B) = \cos(A) * \sin(B) + \sin(A) * \cos(B)$$

$$\sin(A - B) = \sin(A) * \cos(B) - \cos(A) * \sin(B)$$

Complex Multiplication

$$x + y\jmath = \sqrt(x^2 + y^2) * (\cos(\theta) + \sin(\theta)\jmath)$$

$$\theta = \tan^{-1}(\frac{y}{x}), r = \sqrt(x^2 + y^2)$$

Where the magnitude of θ places the angle into the proper quadrant.

$$x + y\jmath = r * \cos(\theta) + r * \sin(\theta)\jmath$$

$$x_1 + y_1\jmath = r_1 * (\cos(\theta_1) + \sin(\theta_1)\jmath)$$

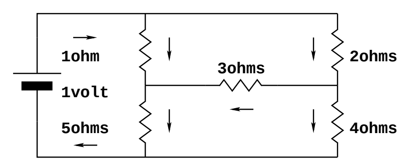
$$x_2 + y_2\jmath = r_2 * (\cos(\theta_2) + \sin(\theta_2)\jmath)$$

$$\begin{aligned}(x_1 + y_1\jmath) * (x_2 + y_2\jmath) = (r_1 * r_2) * \\ (\cos(\theta_1) * \cos(\theta_2) - \sin(\theta_1) * \sin(\theta_2)) + \\ (\cos(\theta_1) * \sin(\theta_2) + \sin(\theta_1) * \cos(\theta_2))\jmath\end{aligned}$$

$$\begin{aligned}(x_1 + y_1\jmath) * (x_2 + y_2\jmath) = \\ r_1 * r_2 * (\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2)\jmath)\end{aligned}$$

In summary, multiply the magnitudes and add the angles in polar form.

Solving for the currents in an unbalanced Whetstone bridge



N.B., this method also works for for steady-state AC computations. The only difference is that the equations are in complex arithmetic.

The currents (I), voltages (V) and resistors (R) are known by their ohms value in this example. In general, these identifications may be assigned arbitarily.

The current flows are in the direction of the small arrows.

Using Kirchhoff's voltage and current laws, 5 independent equations in the 5 unknown currents can be written:

1. The voltage through R_1 and R_5 sums to 1 volt.
2. The voltage through R_2 and R_4 sums to 1 volt.
3. The voltage through R_2, R_3 and R_5 sums to 1 volt.
4. The currents into the junction of R_1, R_3 and R_5 sums to 0.
5. The currents into the junction of R_2, R_3 and R_4 sums to 0.

These equations can be solved by a number of means. I will state the problem in both equation matrix form and not show the inversion of the coefficient matrix. Remember Ohm's Law: $E = I \times R$. Currents and voltage against the arrows are negative.

$$\begin{aligned} 1I_1 + 5I_5 &= 1 \\ 2I_2 + 4I_4 &= 1 \\ 2I_2 + 3I_3 + 5I_5 &= 1 \\ I_1 + I_3 - I_5 &= 0 \\ I_2 - I_3 - I_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 2 & 0 & 4 & 0 \\ 0 & 2 & 3 & 0 & 5 \\ 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Here I am guilty of not showing my work. I solved this matrix equation using a HP 49G programable calculator which has matrix inversion and multiplication built in. The computation took a fraction of a second and easily would have taken a half hour using a basic calculator. Also, I am only showing 5 decimal places.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 0.19355 \\ 0.14516 \\ -0.03226 \\ 0.17742 \\ 0.16129 \end{bmatrix}$$

Note, I guessed the wrong direction for the current in R_3 and that the sum of the currents into R_1 and R_2 equals the sum of the currents out of R_4 and R_5. That total current, 0.33871, indicates that circuit's resistance is 2.9524Ω.

If R_3 wre not present, then the circuit would have had a overall resistance of 3Ω (two parallel 6Ω resistors). R_3 reduced the overall resistance of the circuit.

If $\frac{R_1}{R_5}$ had equaled $\frac{R_2}{R_4}$, then the voltages at the ends of R_3 would have been equal and no current would have flowed through R_3. This is the prrinciple of the Whetstone bridge. Also, in that case, R_3 would have had no effect of the whole circuit's resistance.

Notes on the Log-Trig sheet

Although this sheet has been cleared by the FRRL (W9CEQ) and DARC (W9DUP) VE teams for their sessions, you must verify that the sheet is acceptable with the VE team you are using and plan accordingly. The sheet is available with and without the plain Smith chart and the basic information is on the first page. Either of those adaptions may make the sheet acceptable to your VE team.

From the ARRL VEC's instructions to the VE teams, but the teams *may* be more restrictive than these instructions:

> **Candidates' Possessions**
>
> The candidates must put away any headsets, books, paper or earphones they may have brought with them to the session. Watches that give any kind of alarm or hourly chimes should be disabled so that other candidates won't be distracted during their exams. Candidates who must use hearing aids should be allowed to wear them. Cell phones and pagers should be turned off.
>
> Instruct the candidates that no "crib notes" or other written assistance is permitted once the exams are begun. Using any kind of aid on the test, other than a calculator (see below), is not allowed. (Slide rules and logarithmic tables are acceptable, as long as they're free of notes.) Candidates found using unauthorized aids will have their exams terminated and will be assigned failing grades. They will then be dismissed from the test session.
>
> **Calculators**
>
> Most applicants will bring some type of scientific calculator to use on their exams. Most of these, including some programmable calculators, are acceptable. The candidate must, at the VEs' request, demonstrate that all of the calculator's memories have been cleared. The VE Team has the right to refuse a candidate the use of a calculator if the team isn't convinced that this has been accomplished. (Most calculators clear their memories automatically when they're shut off, but some have an internal backup battery or power source that powers the memories continuously even when the main battery is dead or removed.) Many of today's calculators support programming features, such as built-in formulas.

The log trig tables sheet has been cleared by a number of VE groups as being acceptable "log and trig tables" and only contain information that is either available from tables of trigonometric functions, powers and logarithms or from the Extra class test itself. If the sheet is kept free of **any** additional notes, then you most likely would be permitted to bring it to the Extra test and use it.[1]

If you like, then make an extra copy on which to make your notes and do not bring that annotated copy to the Extra test.

Read the trigonometric function values for 0 to 45 degrees using the labels on the left and top of the table and read the values for 45 to 90 degrees using the labels on the right and bottom of the table.

Because four function calculators are so cheap and available, I did not provide a table of logarithms for doing high precision multiplications and divisions. A set of two-digit argument table of common logarithms and antilogarithms has been provided. The common logarithms table has the first unit digit of the argument in the vertical column and the tenths digit across the horizonal in two rows. E.g., the common logarithm of 7.5 is approximately 0.875. The common antilogaritms table has the tenth digit of the argument in the vertical column and the hundredths digit across the horizonal in two rows. E.g., the common antilogarithm of .88 is approximately 7.586.

[1] Maria Somma AB1FM of the ARRL VEC: "We have a VE team that only allows candidates to use the calculators that they provide and tables are not allowed. Individual teams can decide if they feel comfortable with the candidates having more or less of the allowable aides during the session."

The sine and cosine trigonometric functions are present both in tabular form and as a circular quadrant.

A simple, unlabeled Smith Chart is present in the same form as in these notes. This is permissable as a very similar chart is present in the graphics provided with the Extra class test so that no additional information is provided.

Logarithms of base e and 10 and powers of e and 10 are provided in tabular form and logarithms of base 10 are provided in graphic format. On the left side of each rule is the logarithm base 10 and on the right side of the rule is the decimal value. Using the decimal values, if the left rule is a value, the center rule has the square and the right rule has the cube. In the opposite direction, one can compute square roots and cube roots.

If one multiplies the logarithm by 10, i.e., moves the decimal one place to the right, then the rules can be used look up the multipliers for a decibel conversion.

Algebra Refresher

The mathematical basis of arithematic

Real numbers, complex numbers are normed division algebra fields. There are two others; but, you do not need them for the Extra Class test. Reals ($\mathbb{R}$) measure distances. Complex numbers ($\mathbb{C}$) measure positions on a plane, either on a grid or as a direction and a distance. The integers ($\mathbb{N}$) are not fields; they do not have multiplicative inverses!

I will use $\mathbb{F}$ in rules that apply to both $\mathbb{R}$ and $\mathbb{C}$. $\forall$ means "for all." $\exists$ means "there exists."

Fields have these properties:

Closure The addition or multiplication of two elements of a field produces another element of the field.

$$\forall a, b, c \in \mathbb{F}, a + b = c, a \times b = c$$

Associativity A sequence of the same operation can be done in any pairing.

$$\forall a, b, c \in \mathbb{F}, a + (b + c) = (a + b) + c$$

$$\forall a, b, c \in \mathbb{F}, a \times (b \times c) = (a \times b) \times c$$

Commutivity A sequence of the same operation can be done in any order.

$$\forall a, b \in \mathbb{F}, a + b = b + a$$

$$\forall a, b \in \mathbb{F}, a \times b = b \times a$$

Existence of additive identity

$$\forall a \in \mathbb{F}, a + 0 = a, 0 \in \mathbb{F}$$

Existence of multiplicative identity

$$\forall a \in \mathbb{F}, a \times 1 = a, 1 \in \mathbb{F}$$

Existence of additive inverses

$$\forall a \in \mathbb{F}, \exists -a, a + (-a) = 0 \text{ or } a - a = 0$$

Existence of multiplicative inverses

$$\forall a \neq 0 \in \mathbb{F}, \exists a^{-1} \text{ or } \frac{1}{a} \in \mathbb{F},$$

$$a \times a^{-1} = a \times \frac{1}{a} = \frac{a}{a} = 1$$

Distributivity $\forall a, b, c \in \mathbb{F}, a \times (b + c) = a \times b + a \times c$

What about subtraction and division?

Subtraction is adding the additive inverse of the subtrahed.

$$a - b = a + (-b)$$

Division is multiplying by the multiplicative inverse of the divisor.

$$\frac{a}{b} = a \times \frac{1}{b}$$

Various consequences

$$\frac{(\frac{a}{b})}{c} = \frac{a}{bc}$$

$$\frac{a}{(\frac{b}{c})} = \frac{ac}{b}$$

$$a\frac{b}{c} = \frac{ab}{c}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\text{If } a \neq 0, \text{ then } \frac{ab}{ac} = \frac{b}{c}, \text{ else undefined!}$$

$$\text{If } a \neq 0, \text{ then } \frac{ab}{a} = b, \text{ else undefined!}$$

Working with exponents

$\forall, a \neq 0, b \in \mathbb{F}$

$$a^{c+d} = (a^c) \times (a^d)$$

$$(a^c)^d = a^{c \times d}$$

$$(a \times b)^c = a^c \times b^c$$

$$a^{-c} = \frac{1}{a^c}$$

$a^0 = 1$

$a^{\frac{1}{2}} = \sqrt{a}$

$a^{\frac{1}{c}} = \sqrt[c]{a}$

$e^{\jmath \times \pi} + 1 = 0$

$\jmath^\jmath = e^{-\frac{\pi}{2}} \approx 0.2078795763507619$

If $n \in$ even $\mathbb{N}$, then $(a^n)^{-n} = \sqrt[n]{a^n} = |a|$

The equation $x^3 = 1$ has three roots in $\mathbb{C}$: $1\angle 0^\circ = (\cos 0, \sin 0)$, $1\angle 120^\circ = (\cos 120^\circ, \sin 120^\circ)$ and $1\angle 240^\circ = (\cos 240^\circ, \sin 240^\circ)$.

Working with logarithms

$\forall, a \neq 0, b, c, d \in \mathbb{F}$

$b = \log_a(c)$ is $a^b = c$

$\ln x = \log_e x, e \approx 2.718281828$

$\log x = \log_{10} x$

$|\log_a 0| = \inf$

$|\log_a 1| = 0$

$|\log_a a^b = b$

$|\log_a b \times c = (\log_a b) + (\log_a c)$

$|b \neq 0, \log_a \frac{1}{b} = -\log_a b)$

$\log_a c = \frac{\log_b c}{\log_b c}$

Solving quadratics

$Ax^2 + Bx + C = 0$

$x^2 + B'x + C' = 0, B' = \frac{B}{A}, C' = \frac{C}{A}$

$x = r$ and $x = s \Rightarrow (x - r)(x - s) = 0$

$x^2 + (-(r + s))x + rs = 0$

$r + s = -B'$ and $rs = C'$

$(r + s)^2 = B'^2 = r^2 + 2rs + s^2$

$B'^2 - 4C' => r^2 - 2rs + s^2 \Rightarrow (r - s)^2$

$r - s = \sqrt{B'^2 - 4C'}$

$r = \frac{-B' + \sqrt{B'^2 - 4C'}}{2}$

$s = \frac{-B' - \sqrt{B'^2 - 4C'}}{2}$

$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

if $B^2 - 4AC \geq 0$, then two real roots

if $B^2 - 4AC < 0$, then two complex roots

Inequalities $\forall, a, b, c$

If $a > b$, then $a + c > b + c$

If $a > b$ and $c > 0$, then $a \times c > b \times c$

If $a > b$ and $c < 0$, then $a \times c < b \times c$

Absolute value $\forall a, b \in \mathbb{F}$

Absolute value is the distance from the origin.

$|a| \geq 0$

$|a| = |-a|$

$|a \times b] = |a||b|$

$|a + b| \leq |a| + |b|$

$|a| = \sqrt{a^2}$

If $a \in \mathbb{R}$, then $|a| =$ if $a \geq 0$, then a, else $-a$

Complex arithmetic

Complex arithmetic is discussed on page 4.

The complex conjugate * of $a + b\jmath$ is $a - b\jmath$.

The product $x \times x*$ is $(a + b\jmath) \times (a - b\jmath)$ is $a^2 + b^2$.

Common mistakes

Thou shalt not divide by zero.

Watch your parentheses, particularly the implicit parentheses!

The numerator and denominator of fractions have implicit parentheses.

Legalese

NCVEC Extra Class Question Pool 2012 declared public domain.

http://www.ncvec.org/downloads/REVISED%202012-2016%20Extra%20Class%20Pool.txt

Quotations from the ARRL VE instructions and from a Maria Somma AB1FM of the ARRL VEC e-mail.

Anim8or 0.97d beta, Google SketchUp 8 and Wolfram Mathematica 8 used to create some of the illustrations. Other illustrations are hand written Adobe PostScript language programs.

4 place Logarithms base 10 with deltas

	0	1	2	3	4	5	6	7	8	9	del	1	2	3	4	5	6	7	8	9
10x	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	43	4	9	13	17	22	26	30	34	39
11x	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	42	4	8	13	17	21	25	29	34	38
12x	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	41	4	8	12	16	21	25	29	33	37
13x	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	40	4	8	12	16	20	24	28	32	36
14x	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	39	4	8	12	16	20	23	27	31	35
15x	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	38	4	8	11	15	19	23	27	30	34
16x	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	37	4	7	11	15	19	22	26	30	33
17x	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	36	4	7	11	14	18	22	25	29	32
18x	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	35	4	7	11	14	18	21	25	28	32
19x	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	34	3	7	10	14	17	20	24	27	31
20x	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	33	3	7	10	13	17	20	23	26	30
21x	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	32	3	6	10	13	16	19	22	26	29
22x	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	31	3	6	9	12	16	19	22	25	28
23x	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	30	3	6	9	12	15	18	21	24	27
24x	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	29	3	6	9	12	15	17	20	23	26
25x	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	28	3	6	8	11	14	17	20	22	25
26x	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	27	3	5	8	11	14	16	19	22	24
27x	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	26	3	5	8	10	13	16	18	21	23
28x	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	25	3	5	8	10	13	15	18	20	23
29x	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	24	2	5	7	10	12	14	17	19	22
30x	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	23	2	5	7	9	12	14	16	18	21
31x	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	22	2	4	7	9	11	13	15	18	20
32x	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	21	2	4	6	8	11	13	15	17	19
33x	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	20	2	4	6	8	10	12	14	16	18
34x	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	19	2	4	6	8	10	11	13	15	17
35x	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	18	2	4	5	7	9	11	13	14	16
36x	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	17	2	3	5	7	9	10	12	14	15
37x	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	16	2	3	5	6	8	10	11	13	14
38x	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	15	2	3	5	6	8	9	11	12	14
39x	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	14	1	3	4	6	7	8	10	11	13
40x	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	13	1	3	4	5	7	8	9	10	12
41x	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	12	1	2	4	5	6	7	8	10	11
42x	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	11	1	2	3	4	6	7	8	9	10
43x	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10	1	2	3	4	5	6	7	8	9
44x	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	9	1	2	3	4	5	5	6	7	8
45x	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618										
46x	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712										
47x	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803										
48x	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893										
49x	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	8	1	2	2	3	4	5	6	6	7
50x	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067										
51x	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152										
52x	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235										
53x	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316										
54x	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396										
55x	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	7	1	1	2	3	4	4	5	6	6
56x	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551										
57x	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627										
58x	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701										
59x	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774										
60x	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846										
61x	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917										
62x	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987										
63x	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	6	1	1	2	2	3	4	4	5	5
64x	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122										
65x	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189										
66x	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254										
67x	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319										
68x	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382										
69x	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445										
70x	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506										
71x	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567										
72x	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627										
73x	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	5	1	1	2	2	3	3	4	4	5
74x	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745										
75x	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802										
76x	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859										
77x	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915										
78x	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971										
79x	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025										
80x	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079										
81x	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133										
82x	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186										
83x	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238										
84x	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289										
85x	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340										
86x	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390										
87x	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440										
88x	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	4	0	1	1	2	2	2	3	3	4
89x	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538										
90x	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586										
91x	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633										
92x	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680										
93x	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727										
94x	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773										
95x	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818										
96x	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863										
97x	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908										
98x	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952										
99x	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996										